

# Sets, Permutations and Combinations

Note Title

24/09/2008

Conventionally, sets are denoted using curly brackets:

{ Monday, Tuesday, Wednesday, Thursday, Friday }

{ 0 }

{ red, yellow, blue }

## The Membership Relation

$x \in S$  means  $x$  is a member of  
(or element of) set  $S$ .

$\in$  denotes the membership relation.

The number of elements in a set  $S$  is denoted by  $|S|$ .

E.g.

{ Monday, Tuesday, Wednesday, Thursday, Friday }

{ 0 }

{ red, yellow, blue }

## Cartesian Product

The cartesian product of sets  $S$  and  $T$  is denoted  $S \times T$ .

The elements of  $S \times T$  are ordered pairs  $(s, t)$  where  $s \in S$  and  $t \in T$ .

E.g.

## Permutations

A *permutation* of a set is a way of arranging the elements of the set in order.

Eg.  $\{a, b\}$  has two permutations:  $a b$  and  $b a$ .

$\{a, b, c\}$  has six permutations:

$a b c$	$b a c$	$c a b$
$a c b$	$b c a$	$c b a$

## Subsets

Set  $S$  is a subset of set  $T$ , written  $S \subseteq T$ , if every element of  $S$  is an element of  $T$ .

E.g.

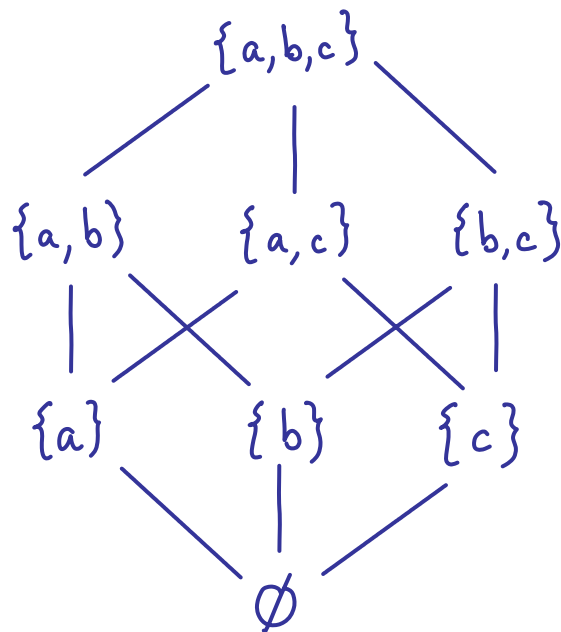
$$\{a\} \subseteq \{a, b\}$$

$$\{\text{red, green}\} \subseteq \{\text{red, yellow, green, blue}\}$$

$\emptyset$  denotes the empty set.

$$[\emptyset \subseteq S]$$

## Hasse Diagram of Subset Relation on $2^{\{a,b,c\}}$



## Combinations

$nCr$  is the number of subsets of cardinality  $r$  of a set of cardinality  $n$ .

- the number of ways of choosing  $r$  elements from a set with  $n$  elements.

## Permutations

$nPr$  is the number of ways of choosing  $r$  elements from a set of  $n$  elements *in order*. (I.e. choosing the 1st, the 2nd, ..., the  $r$ th.)

6 people are at the left bank of a river.  
They want to get 3 of them across to the right bank.  
Their boat will only carry 2 of them at a time.  
How many different ways can this be done?